ACOUSTIC FLOWS AROUND A SPHEROID

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UDC 534.222.2

It is known that stationary vortex flows (acoustic flows) can occur in the field of a sound wave around different obstacles. This phenomenon was investigated in a number of papers, part of which is examined in the surveys [1-3], where the most intent attention was paid to flows near bodies of regular geometric shape (plane, tube, cylinder, sphere) in a standing wave field without taking account of heat conduction. The influence of heat conduction on the origination of flows in a boundary layer under oblique incidence of a travelling sound wave on a plane is examined in [4].

The acoustic flows in a boundary layer originating in a real medium under the action of a plane travelling sound wave incident along the axis of rotation of a prolate spheroid are determined in this paper.

The solution for the acoustic flow velocity can be obtained from the fundamental equations of motion of a viscous fluid by successive approximations. It has the form [1]

$$\mu\nabla^2\mathbf{V}_2 = \nabla p_2 - \mathbf{F}; \tag{1}$$

$$\mathbf{F} = -\rho_0 \langle(\mathbf{V}_1\nabla)\mathbf{V}_1 + \mathbf{V}_1\nabla\mathbf{V}_1\rangle, \tag{2}$$

where $\mathbf{V}_2$ is the acoustic flow velocity, p_2 is a small correction to the pressure in the sound field, is the force, averaged with respect to time, that is found in a field of first order of smallness $\mathbf{V}_1$, corresponding to the solution of the equations of motion of a viscous heat conducting fluid in a linear formulation, ρ_0 is the density of the medium not perturbed by sound, and μ is the dynamic viscosity coefficient.

Let a plane sound wave with the velocity potential ψ_p (Fig. 1) be incident along the axis of rotation of a prolate absolutely stiff spheroid. We assume the spheroid conducts heat perfectly. Let us examine the case when the wavelength is much less than the spheroid dimensions. Let us determine the expression for $\mathbf{V}_1$. We will seek the solution in the boundary layer of the spheroid where the influence of the viscous and thermal waves is most substantial. Only the exposed domain is subject to study since under our assumption there is no scattered field in the shadow.

The complete system of equations of motion of a continuous medium describing small perturbations for the steady vibration mode with time dependences $\exp(-i\omega t)$ can be reduced to a system of Helmholtz equations [5]

$$\Delta\psi_1 + k_{11}^2\psi_1 = 0,\quad \Delta\psi_2 + k_{12}^2\psi_2 = 0,\quad \Delta\mathbf{\Phi} + k_2^2\mathbf{\Phi} = 0, \tag{3}$$

where ψ_1, ψ_2, $\mathbf{\Phi}$ are, respectively, the longitudinal, thermal, and viscous wave potentials;

$$\mathbf{V}_1 = \nabla\psi + \nabla\times\mathbf{\Phi}. \tag{4}$$

Here $\psi = \psi_1 + \psi_2$; $\psi_1 = \psi_p + \psi_s$; ψ_s is the scattered wave potential. In the case when the viscosity and heat conduction coefficients are small, $k_{11} \approx \omega/c$, $k_{12} \approx \alpha(1 + i)$, $k_2 \approx \beta(1 + i)$, $\alpha = \sqrt{\omega/2\varkappa_0}$, $\beta = \sqrt{\omega/2\nu}$ (ω is the vibration frequency, c is the sound speed, $\varkappa_0$ is the thermal diffusivity coefficient, and ν is the kinematic viscosity coefficient). We seek the solution of the system (3) by the local field method by assuming that the acoustic field of a small neighborhood of a surface point depends only on the incident wave and the geometric shape of the section of the surface. We select the neighborhood of a surface point M such that the thickness of the domain δ would be much less than its lateral dimensions $\tau(\delta \ll \tau)$. Here $\delta = \max\{\alpha^{-1}; \beta^{-1}\}$. Let us use a local curvilinear orthogonal system of coordinates u, v, w. We then write the system (3) in the form

Tula. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 99-102, November-December, 1988. Original article submitted August 19, 1987.

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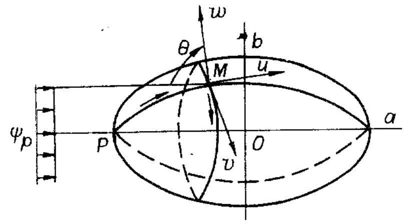

Fig. 1

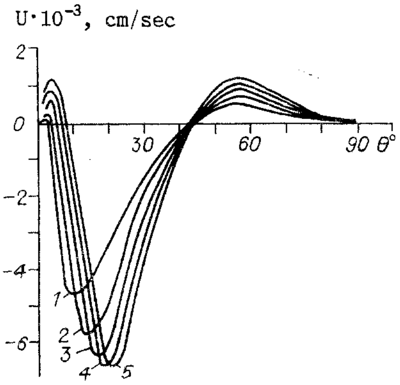

Fig. 2

$$\frac{\partial^2\psi_1}{\partial u^2} + \frac{\partial^2\psi_1}{\partial w^2} + (\varkappa_u + \varkappa_v)\frac{\partial\psi_1}{\partial w} + k_{11}^2\psi_1 = 0,$$

$$\frac{\partial^2\psi_2}{\partial w^2} + (\varkappa_u + \varkappa_v)\frac{\partial\psi_2}{\partial w} + k_{12}^2\psi_2 = 0, \qquad (5)$$

$$\frac{\partial^2\Phi}{\partial w^2} + (\varkappa_u + \varkappa_v)\frac{\partial\Phi}{\partial w} + k_2^2\Phi = 0,$$

where $\varkappa_u = (ab)^{-2}(a^2\cos^2\theta + b^2\sin^2\theta)^{3/2}$ and $\varkappa_v = (a^2\cos^2\theta + b^2\sin^2\theta)^{1/2} \times (b^2\sin^2\theta)^{-1}$ are the curvatures of points of the surface.

The symmetry of the problem ($\psi_1 = \psi_1(u, w)$, $\psi_2 = \psi_2(u, w)$, $\mathbf{\Phi} = \Phi(u, v)\mathbf{e}_V$) as well as the fact that the sound wavelength was selected as the characteristic length l ($l \gg \tau$) were taken into account in deriving the system (5) and all terms of the order of $(\delta/l)^2$ were discarded.

The boundary conditions for the solution of (5) have the form

$$V_{1u} = \frac{\partial\psi}{\partial u} - \frac{\partial\Phi}{\partial w} - \varkappa_v\Phi\Big|_{w=0} = 0,$$

$$V_{1w} = \frac{\partial\psi}{\partial w} + \frac{\partial\Phi}{\partial u}\Big|_{w=0} = 0, \quad T = \frac{1}{\alpha'}\left(\frac{i\omega\gamma}{c^2}\psi + \frac{i}{\omega}\Delta\psi\right)\Big|_{w=0} = 0. \qquad (6)$$

Here T is the acoustic temperature of the medium, α' is the coefficient of temperature expansion of the medium, and $\gamma = c_p/c_V$.

We seek the solution of (5) in the form of plane waves. If the incident wave potential in the local coordinates is $\psi_p = \exp[i(k_1 u\sin\theta - k_1 w\cos\theta - \omega t)]$, where $k_1 \approx k_{11} - (i/2)(\varkappa_u + \varkappa_v)\cos\theta$, then valid for the scattered wave potentials is $\psi_s = A\exp[i(k_{11u}u + k_{11w}w - \omega t)]$, $\psi_2 = B\exp[i(k_{12u}u + k_{12w}w - \omega t)]$, $\Phi = C\exp[i(k_{2u}u + k_{2w}w - \omega t)]$ [A, B, C are the wave amplitudes found from the boundary conditions (6)]. Here, according to the Snell law on the equality of the phase rates of the incident and scattered waves, $k_{11u} = k_{12u} = k_{2u} = k_1\sin\theta$ and the expressions $k_{11w} = k_{11}\cos\theta + (i/2)(\varkappa_u + \varkappa_v)(1 + \sin^2\theta)$, $k_{12w} = k_{12} + (i/2)(\varkappa_u + \varkappa_v)$, $k_{2w} = k_2 + (i/2)(\varkappa_u + \varkappa_v)$ satisfy the dispersion equations corresponding to the system (5).

It should be noted that this solution is not suitable at the pole P of the spheroid where $\varkappa_v$ becomes infinite. Now, by finding $\mathbf{V}_1$ from (4), it is possible to determine $\mathbf{V}_2$. Using the fact that we seek the solution of the problem in a thin boundary layer, we will neglect the terms ∇p_2 in (1), as is done in [4]. After finding the force $\mathbf{F}$ from (2) we see that its tangential component is much greater than the normal ($F_u \gg F_w$) as it should be according to boundary layer theory [1-4]. Then $\mathbf{F} = F_u(u, w)\mathbf{e}_u$ ($\mathbf{e}_u$ is the direction of the coordinate axis u). Writing the flow velocity $\mathbf{V}_2 = V_2(u, w)\mathbf{e}_u$ is valid analogously, and exactly as the force, will be directed along the u axis.

Taking account of the above-mentioned assumptions, we integrate the system of equations (1) and (2) with the boundary conditions $V_2|_{W=0} = 0$, $\partial V_2/\partial w|_{W\to\infty} = 0$. Then we obtain for the acoustic flow velocity

$$V_2 = f \exp\,[(\varkappa_u + \varkappa_v)\sin\theta\cos\theta u]\{\exp\,[\beta(i-1)w] - 1\} + c.c., \tag{7}$$

where c.c. denotes the complex conjugate, $f = \beta^2 C\left[\left(k_{11} + i\frac{\varkappa_u + \varkappa_v}{2}\cos\theta\right)\cos\theta + \bar{A}\left(k_{11}\cos\theta - i\frac{\varkappa_u + \varkappa_v}{2}\right)\right.$ $\left.(1+\sin^2\theta)\right] \Big/ \{\nu\beta(i-1)\ [\beta(i-1) + \varkappa_u + \varkappa_v]\}$.

Let us note that the value found for V_2 is the mean velocity over several periods that the particles of the medium passing through a given point have, i.e., the velocity from the Euler point of view. In fact, we deal in experiments with the particles themselves and the velocity here is understood in the Lagrange conception. Consequently, to determine the mass flux the transport velocity $\mathbf{V}_T = \langle(\xi_1\nabla)\mathbf{V}_1\rangle$ is introduced (ξ_1 is the displacement vector of particles in the sound wave, $\xi_1 = \int \mathbf{V}_1 dt$). In our case

$$\mathbf{V}_T = \frac{1}{i\omega}\exp\,[(\varkappa_u + \varkappa_v)\sin\theta\cos\theta u]\ \left[\left(k_{11} - i\frac{\varkappa_u + \varkappa_v}{2}\cos\theta\right)\sin\theta + \right.$$
$$\left. + A\left(k_{11}\cos\theta + i\frac{\varkappa_u + \varkappa_v}{2}\right)(1+\sin^2\theta)\right]\beta^2\bar{c}\exp\,[-\beta(i+1)w] + c.c. \tag{8}$$

The upper bar denotes the complex conjugate.

Therefore, the mean velocity of the mass flux is $\mathbf{U} = \mathbf{V}_2 + \mathbf{V}_T$. The acoustic flow velocities were computed by means of the analytic expressions (7) and (8) obtained for different spheroid configurations and sound wave frequencies when air is the containing medium. The results of the numerical computations are presented in Figs. 2 and 3 (the incident wave frequency is 1 and 5 MHz, respectively), where the distribution of the velocity $\mathbf{U}$ is shown at distances of one viscous wavelength from the surface as a function of the angle θ between the normal of the spheroid and the direction of wave incidence. The curves 1-5 are constructed for spheroids with minor semi-axis 1 cm and major semi-axes 1, 2, 3, 4, 5 cm. As is seen from the graphs, the flow velocity grows considerably as the incident wave frequency increases. Vortex displacement towards the axis of rotation occurs as the prolateness of the spheroid increases. Analogous qualitative results are obtained for an elliptic cylinder in [6].

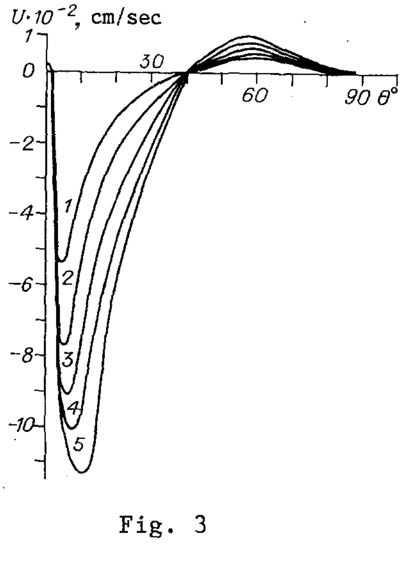

Fig. 3

Let us note that (7) and (8) are obtained with error $(\delta/l)^2$. Here $(\delta/l)^2 = \left[\sqrt{\frac{2\nu}{\omega}}\Big/\frac{2\pi c}{\omega}\right]^2 = \frac{\nu\omega}{2\pi^2 c^2}$.

This error would be of the order of 10^{-6} for the sound wave frequency values and the characteristics of the containing medium selected for the numerical computations. The computations were performed on the electronic computer ES 1033 with the same accuracy.

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